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ProtaStructure Design Guide

Verifying Column Design using Axial Force to Moment (N-M) Interaction Diagram

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Column Interaction Diagram

Objective

The purpose of this document is to outline the steps to verify reinforced concrete column design using the capacity curve, or commonly referred to as N-M (Axial-Moment) Interactive Diagram.

The N-M Interaction Diagram will be derived using empirical formulas of concrete stress block. This is then compared to the same capacity curve generated by ProtaStructure column design to verify the validity and adequacy of its column design.

In fact, this method can be used to check and verify the adequacy of any column design program, as the capacity curve generated by any column design application should tally with the empirical curve as it is derived from first principles.

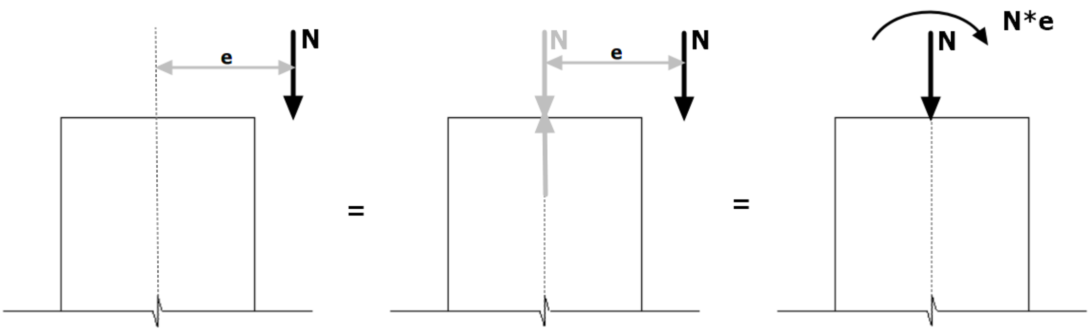
Introduction

Reinforced columns and walls are subjected to combined axial loads and bending moments as part of a 3D building frame behaviour. For reinforced concrete building, the column and beam joints should be assumed as fixed due to monolithic construction. Hence, all columns and walls will be subjected to both axial and moment forces because of gravity as well as lateral external loads.

A capacity of a reinforced concrete column is dependent on the coincident axial load and the moment, i.e. they are not mutually exclusive. An interactive diagram is a capacity curve that can be plotted to completely describe the strength of the column in terms of its axial and moment capacity. Hence, it's common to use the term capacity curve & N-M interaction diagram interchangeably.

Therefore, an interaction diagram can be used to check against the design load combinations and determine the acceptable moment and axial capacities of a structural member.

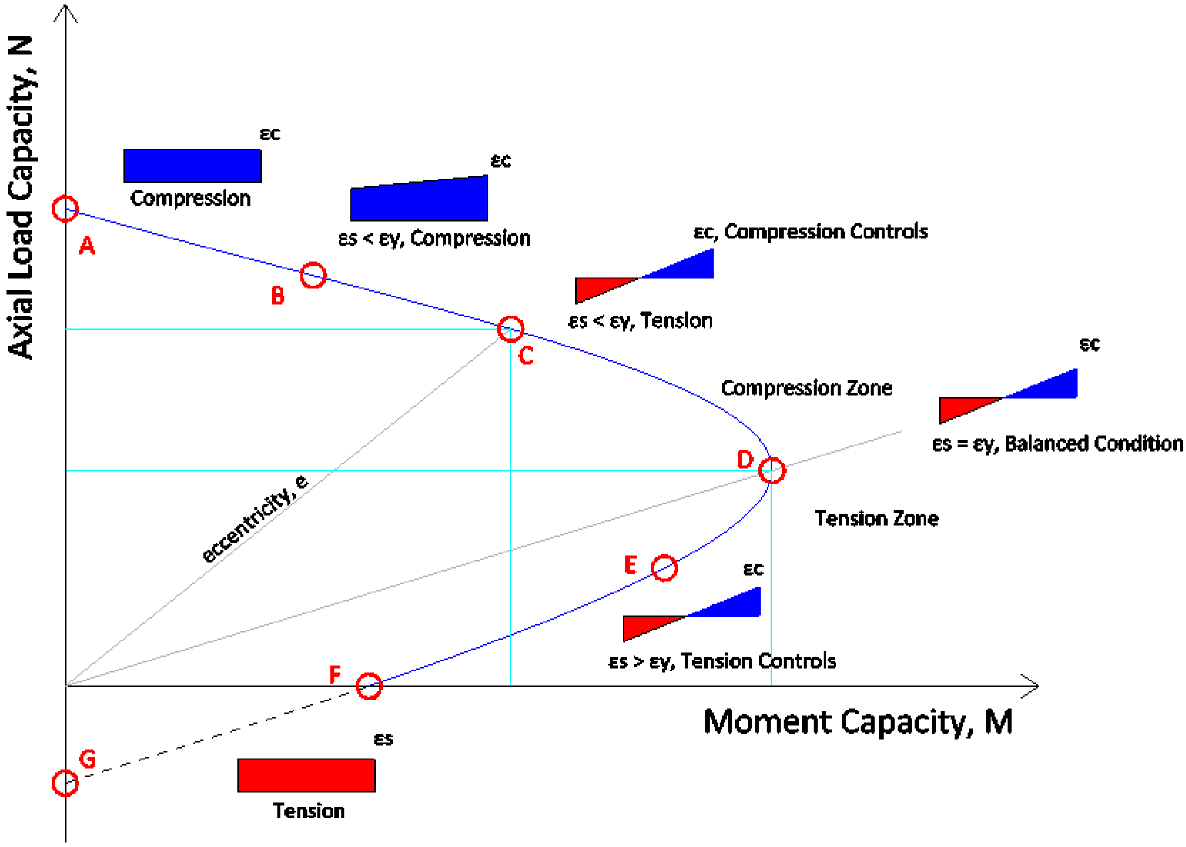
If axial load N is applied with eccentricity, e (distance away from centroid), it creates a pure moment, N*e and to be combined with the original axial load, N.



As the eccentricity, e changes from the section centroid, behaviours of a column/wall as well as stress distribution in the section will be generated differently. These are used to plot the Axial Load to Moment (N–M) Interaction curve.

In reinforcement concrete building, moments are naturally generated due to fixity of the joints even under gravity load, not necessarily due to eccentric eccentricity alone.

Axial Load to Moment (N-M) Diagram Explained:



Case A: Pure axial compression

- The column can only achieve this maximum compression capacity if there is zero co-incident moment. In reality, this capacity can never be achieved as small moment will exist.

Case B: Compression with minor bending due to small eccentricity.

- The axial capacity starts to decrease with increase in moment. Columns at ground level are likely to be at this part of the curve as it is subject to high axial load.

Case C: Compression controls

- The axial capacity decreases rapidly with increasing moment.
- Column subjected to compression and tension whereas the stress in steel at tension side, fst is less than the yield stress, fyk.

Case D: Balanced Condition

- This is the maximum moment capacity, provided the coincident axial value is exactly value D.
- Compression concrete strain, εc reaches maximum strain limit and steel strain, εs is equivalent to steel strain limit, εy.

Case E: Tension controls.

- Both the axial & moment capacity decreases after the balanced point.
- Example, a roof column will likely be at this part of the curve due to low axial loads. Hence, roof columns have lower moment capacity, compared to column at ground level.
- Column subjected to compression and tension whereas the stress in steel at tension side, fst is greater than the yield stress, fyk.

Case F: Pure Flexure

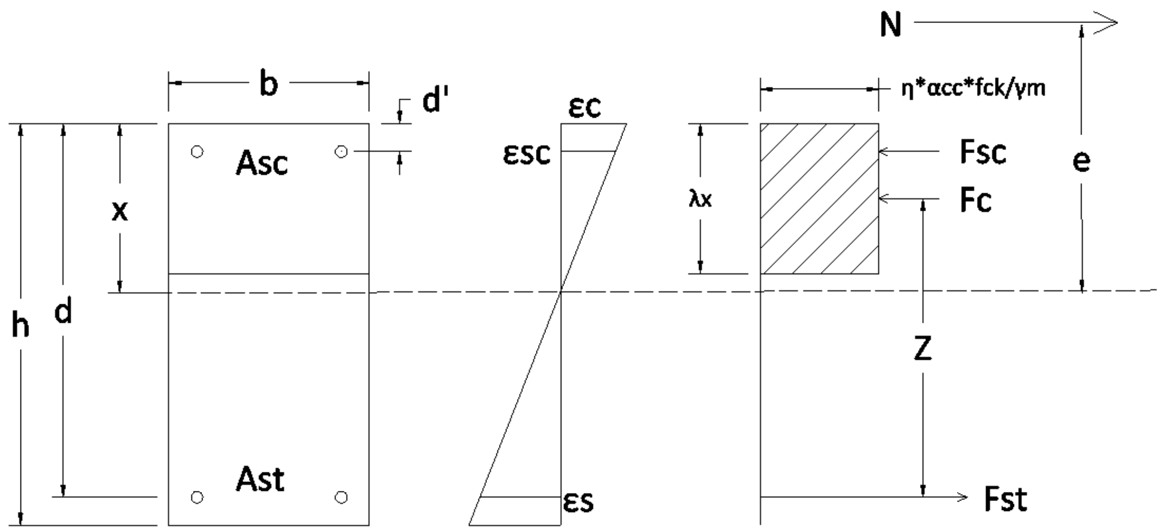
- The column is subjected to pure moment only & zero axial force.

Case G: Pure axial tension

- Column subjected tension force only.
- This case happens to buildings which is subjected to rotational force where some of the columns are in compression, and some are in tension. In reality, this capacity can never be achieved as small moment will exist.

Stress Block

All formulas will be extracted and derived from this stress block.



Notation Based on the Stress Block

Symbols	Definition
Asc	Reinforcement area at extreme compression side
Ast	Reinforcement area at extreme tension side
b	Member width
d	Member depth
d’	Distance from the compression face to center of the first layer rebars
e	Eccentricity
Fc	Force at compressive side by concrete
Fsc	Force at compressive side by steel
Fst	Force at tension side by steel
fyk	Rebar steel grade
h	Member height
N	Axial force
x	Distance between compression face to neutral axis
Z	Lever arm of Internal Forces, distance from Fc to Fst
αcc	Coefficient - eqn. 3.15
εc	Strain in concrete at compressive side
εs	Strain in steel at tension side
εsc	Strain in steel at compressive side
λ	Reduction Factor
η	Effective Strength
ϒm	Concrete Modification Factor

Formula:

Reinforcement area at extreme tension side:

$$Ast = no.bar * \frac{\pi * \varphi^2}{4}$$

Reinforcement area at extreme compression side:

$$Asc = no.bar * \frac{\pi * \varphi^2}{4}$$

Strain in steel at Yield Strength:

$$\varepsilon y = \frac{fyk}{Es} = 0$$

Strain in steel at tension side:

$$\varepsilon_s = \varepsilon_c * \left(\frac{d - x}{x}\right)$$

Strain in steel at compression side:

$$\varepsilon_{sc} = \varepsilon_c * (\frac{x - d'}{x})$$

Tensile stress at tension side:

$$fs = \varepsilon_s * Es$$

Tensile stress at compression side:

$$fsc = \varepsilon_{sc} * Es$$

Compressive stress block depth:

$$Eurocode\ 2\ cl.\ 3.1.7(3):\ \lambda x$$

Forces at compression side:

By concrete: $Fcc = fcd * (RF * \lambda x) * b * \eta$

By steel: $Fsc = Asc * [\max ((min(fy, \varepsilon sc * E), -fyd) - (fcd)]$

Force at tension side:

By steel: $Fst = Ast * min(fy, \varepsilon st * E)$

Summation of forces:

$$\sum F = 0;$$
$$Fst + N = Fcc + Fsc$$
$$N = Fcc + Fsc - Fst$$

Summation of Moment about F_{st}:

$$N * e * (\frac{h}{2} - d') - Fcs * (d - d') - Fcc * (d - \frac{x}{2}) = 0$$

Eccentricity about F_{st}:

$$E = \frac{Fcs * (d - d') + Fcc * \left(d - \frac{\lambda x}{2}\right)}{N} - (h/2 - d')$$

Ultimate Moment Capacity:

$$M = N * e$$

Ultimate Moment Capacity Limit (**British Standard Only – BS8110-1, eqn. 38**):

$$N = 0.4 * fcu * Ac + 0.75 * As * fyd$$

Example (Eurocode):

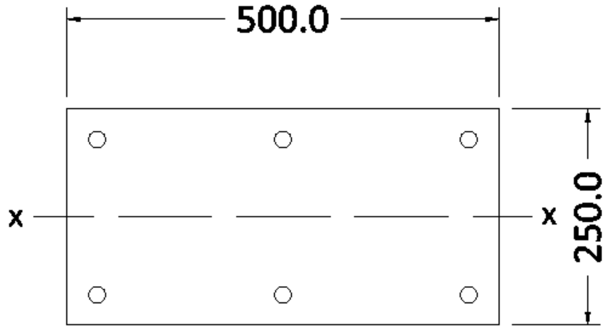
Take the column 1C5 as an example from **"Quick Start Guide Concrete Complete"**.

Column 400mm x 200mm, concrete grade C30/37, steel grade 500 (Type 2),

Main Bar diameter= H13, link bar diameter= H10, cover = 35mm.

Take the first 3 design loads from Design Report in Interactive Design, and design based on N and M22.

Case	Axial Load	Without imperfection and second order effect	With imperfection and second order effect
	N (kN)	M22 (kNm)	**M22 (kNm)**
1	**999.47**	23.7	**28.04**
2	**960.04**	24.6	**28.78**
3	**848.89**	15.1	**18.83**



Factor:	Symbol:	Value:
Coefficient - eqn. 3.15	αcc	0.85
Effective Strength	η:	1.0
Reduction Factor	λ:	0.8
Reduction Factor - cl.3.1.7(3)	RF:	0.9
Max. Strain of Steel	εc:	0.0035
Steel Modulus	Es:	200,000 N/mm^2
Concrete Modification Factor	γm:	1.5
Rebar Modification Factor	γms:	1.15

Plastic Centroid, yp = 200mm

Calculation:

Case G: Pure Tension from Steel

$$Total\ Axial\ Load, N = -Ast * fyd$$
$$N = -346.25kN$$

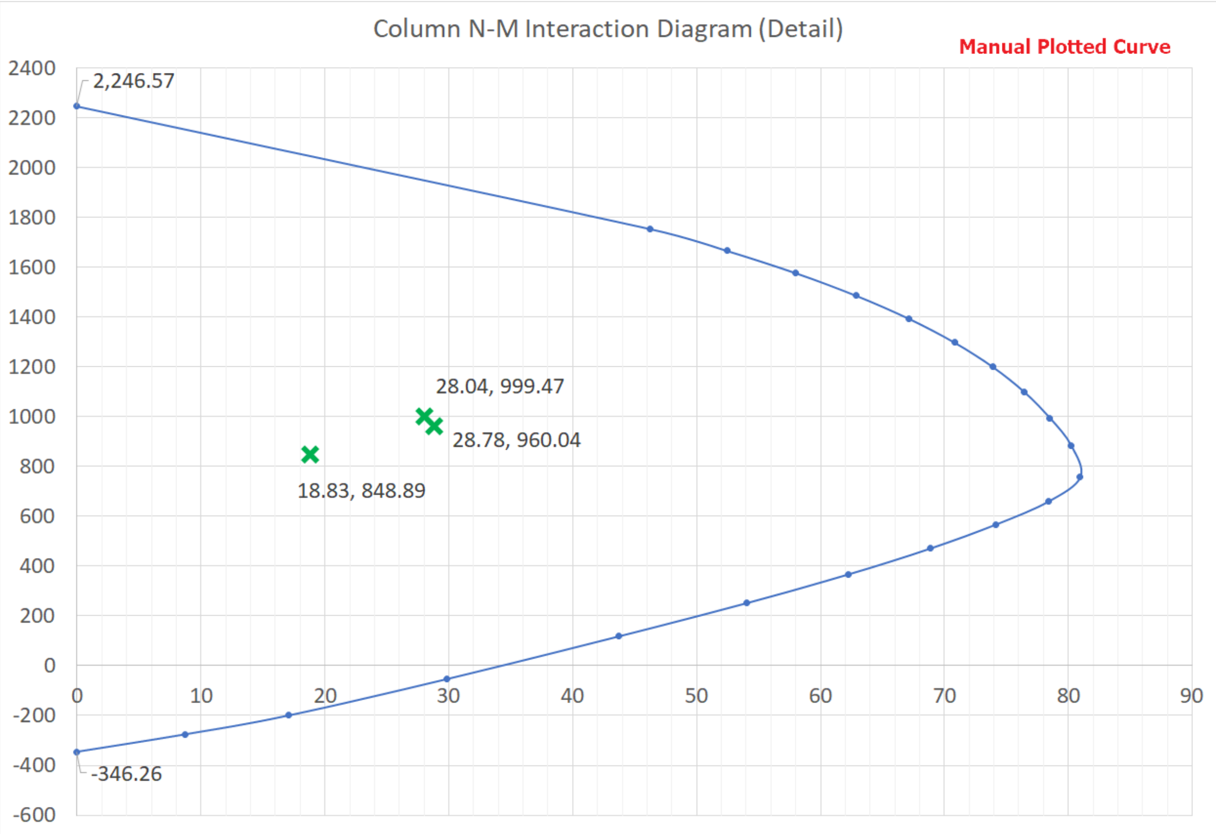
Case A: Pure Compression:

$$Total\ Axial\ Load, N = Fsc + Fcc$$
$$N = (346.25 + 1{,}900.32)kN$$
$$N = 2{,}246.57\ kN$$

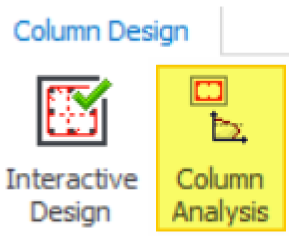
The sample size for x is 20 in this example, increase the size to have higher precision of the result.

$$x\ (mm) = \begin{pmatrix} 20 \\ 40 \\ 60 \\ 80 \\ 100 \\ 120 \\ 140 \\ 160 \\ 180 \\ 200 \\ 220 \\ 240 \\ 260 \\ 280 \\ 300 \\ 320 \\ 340 \\ 360 \\ 380 \\ 400 \end{pmatrix} \quad N, ult\ (kN) = \begin{pmatrix} -276.53 \\ -200.03 \\ -54.46 \\ 117.74 \\ 251.66 \\ 366.44 \\ 470.28 \\ 567.29 \\ 659.74 \\ 758.23 \\ 884.20 \\ 994.24 \\ 1099.11 \\ 1199.93 \\ 1297.51 \\ 1392.45 \\ 1485.22 \\ 1576.19 \\ 1665.63 \\ 1753.78 \end{pmatrix} \quad M, ult\ (kNm) = \begin{pmatrix} 8.68 \\ 17.10 \\ 29.82 \\ 43.74 \\ 54.08 \\ 62.25 \\ 68.85 \\ 74.18 \\ 78.42 \\ 80.97 \\ 80.22 \\ 78.52 \\ 76.44 \\ 73.88 \\ 70.80 \\ 67.15 \\ 62.90 \\ 58.01 \\ 52.47 \\ 46.26 \end{pmatrix}$$

Based on the output from manually potted curve say using Excel program, as shown below.



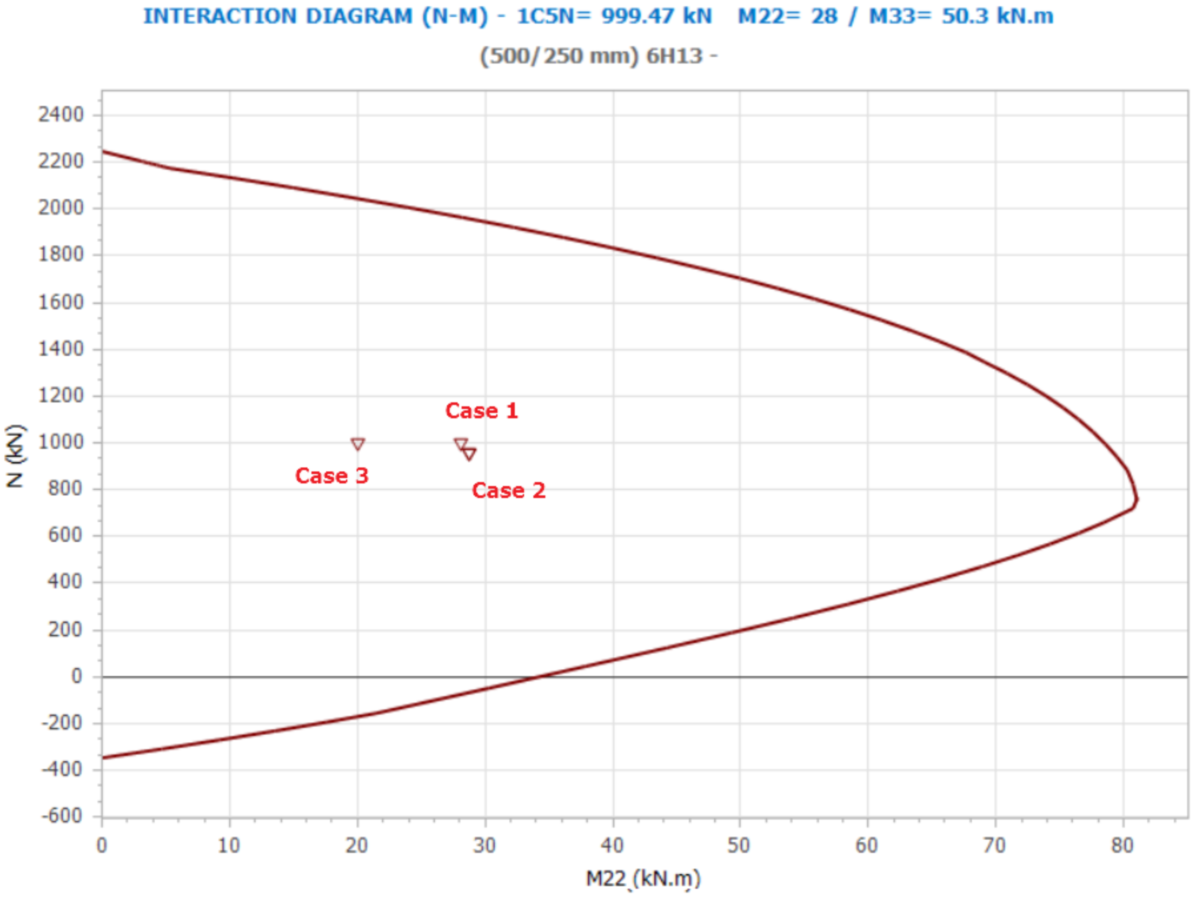
In ProtaStructure, the interaction diagram can be displayed in "Column Analysis" in the Interactive Column design after a successful design (as shown below).



Design Loads: **Design Report from Interactive Design**

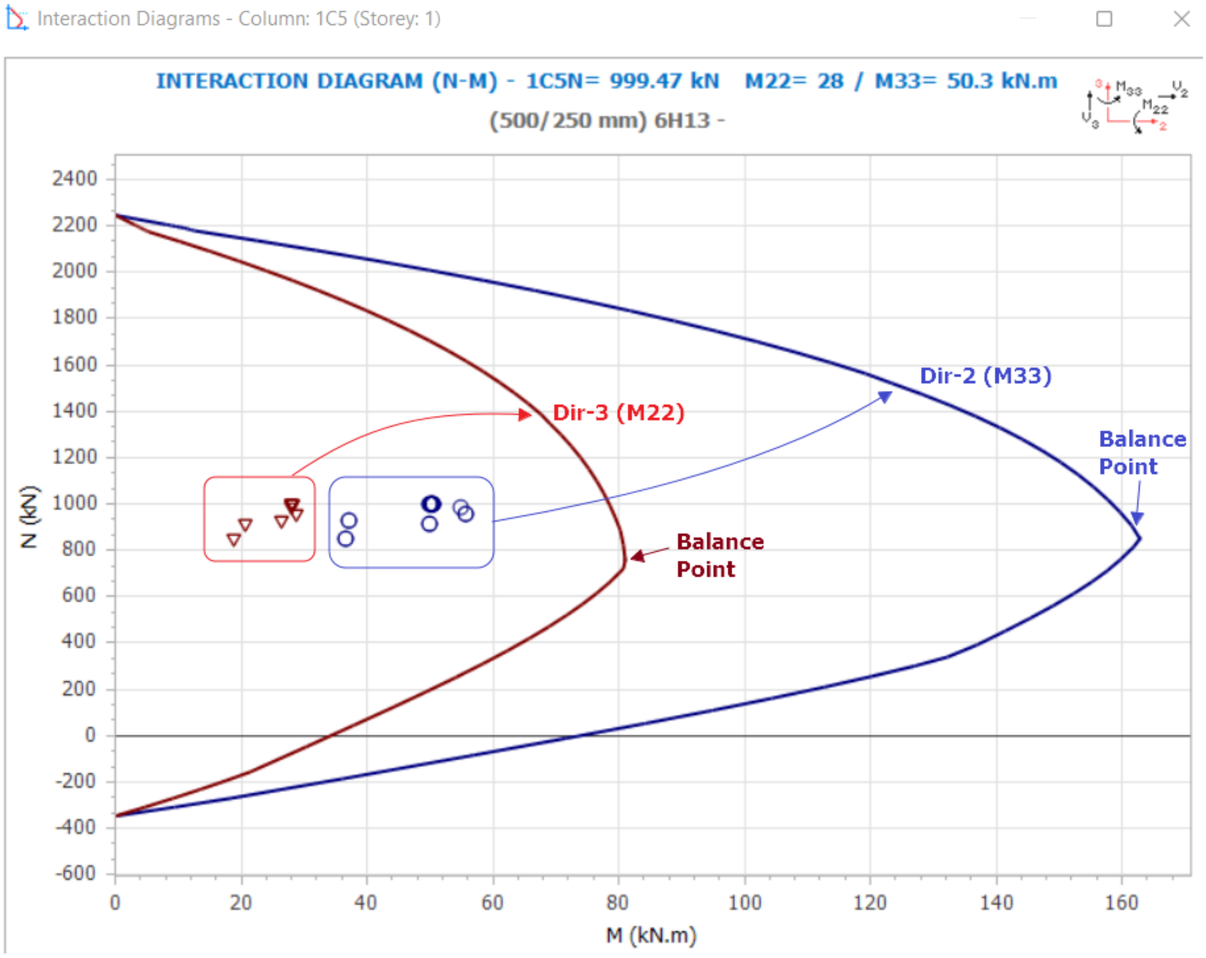
No	N (kN)	M22 (kN.m)
1	999.47	28.04
2	960.04	28.78
3	848.89	18.83

Curve for direction 3 (M22) about “plane X” from ProtaStructure is shown below:



The interaction diagram generated is identical to the manually plotted diagram derived using empirical formulae. Hence, we can conclude that the N-M interaction Diagram result in ProtaStructure is correct and adequate.

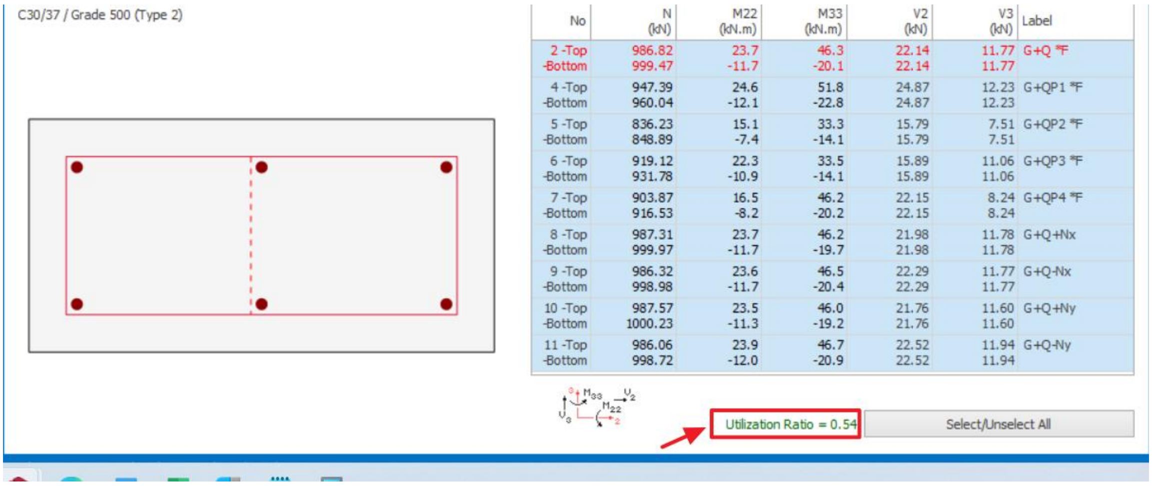
Curve for both direction 2 (M33) & 3 (M22) is shown below:



Both direction (2 & 3) can be verified using the same formulae. When the design loads fall within the curve, the column section size and applied steel rebars are adequate. The governing design will be the direction with load combinations closer to the curve. For this case will, the governing curve is the dir-3 (M22). User may refer to the curve and choose to reduce/increase the rebars size and/or column section size for each direction. If any of the design loads from either direction surpassed the curve, the column design fails.

Column Utilisation Ratio

After “Interactive Design” has been performed, the Utilisation Ratio will be shown at the bottom of the dialogue. This utilisation ratio shows the efficiency design of the column.



No	N (kN)	M22 (kN.m)	M33 (kN.m)	V2 (kN)	V3 (kN)	Label
2 -Top	986.82	23.7	46.3	22.14	11.77	G+Q *F
-Bottom	999.47	-11.7	-20.1	22.14	11.77	
4 -Top	947.39	24.6	51.8	24.87	12.23	G+QP1 *F
-Bottom	960.04	-12.1	-22.8	24.87	12.23	
5 -Top	836.23	15.1	33.3	15.79	7.51	G+QP2 *F
-Bottom	848.89	-7.4	-14.1	15.79	7.51	
6 -Top	919.12	22.3	33.5	15.89	11.06	G+QP3 *F
-Bottom	931.78	-10.9	-14.1	15.89	11.06	
7 -Top	903.87	16.5	46.2	22.15	8.24	G+QP4 *F
-Bottom	916.53	-8.2	-20.2	22.15	8.24	
8 -Top	987.31	23.7	46.2	21.98	11.78	G+Q+Nx
-Bottom	999.97	-11.7	-19.7	21.98	11.78	
9 -Top	986.32	23.6	46.5	22.29	11.77	G+Q-Nx
-Bottom	998.98	-11.7	-20.4	22.29	11.77	
10 -Top	987.57	23.5	46.0	21.76	11.60	G+Q+Ny
-Bottom	1000.23	-11.3	-19.2	21.76	11.60	
11 -Top	986.06	23.9	46.7	22.52	11.94	G+Q-Ny
-Bottom	998.72	-12.0	-20.9	22.52	11.94	

This utilisation ratio will be based on 3 checks and shown only the governed check. The checks can be found in the “Design Report” in the “Interactive Design”. The 3 checks are:

1. Required steel area over provided steel area.
2. Design axial Load, Nd, over maximum axial load capacity of the column.
   (This check only applies to EC8 and BS8110.)
3. Design moment, Md, over maximum moment capacity, Msup, of the column.
   a. Md = Design moment
   b. Msup = Maximum column moment capacity

The formula to calculate Md and Msup are as below:

$$Md = \sqrt{(design\ M22)^2 + (design\ M33)^2}$$

$$Md = \sqrt{50.3^2 + 28^2} = \mathbf{57.57kNm}$$

$$Msup = \sqrt{(max\ M22)^2 + (max\ M33)^2}$$

$$Msup = \sqrt{93^2 + 52^2} = \mathbf{106.6kNm}$$


```
UTILIZATION RATIOS

    As (Required/Supplied)          = 125.00 / 796.39                = 0.16
    N  (Nd)                         = 999.47
    M (Md/Msup)                     = 57.55 / 106.80                 = 0.54
```

Based on 3 utilisation ratios of the column 1C5, Md/Msup ratio has the highest figure, hence, we can conclude that the column is governed by moment.

Example (British Standard):

Another quick and simple example for BS8110 reinforced concrete column is shown here.

Column 400mm x 400mm, concrete grade 30, steel grade 460 (Type 2),

Main Bar diameter= T12, link bar diameter= T10, cover = 30mm.

Apply dead load 180kN vertically and 15kN horizontally (dir-y) at the top of the column.

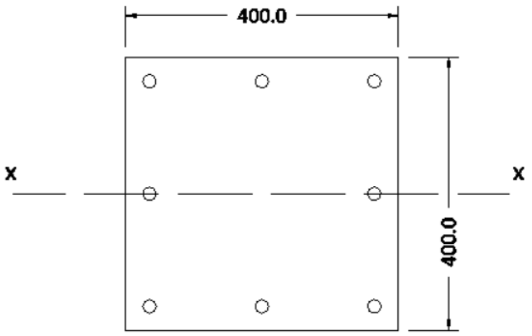
Take the random 3 design loads from Design Report in Interactive Design, and design based on N and M22.

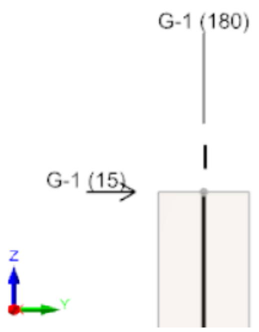
No	N (kN)	M22 (kN.m)
1 -Top	252.0	0.0
-Bottom	268.1	63.0
2 -Top	180.0	0.0
-Bottom	196.1	45.0
6 -Top	216.0	0.0
-Bottom	229.8	62.6

Design Loads:

No	N (kN)	M22 (kN.m)
1	268.13	68.00
2	196.13	48.73
6	229.82	66.95

Case	Axial Load	Without imperfection and second order effect)	With imperfection and second order effect
	N (kN)	M22 (kNm)	**M22 (kNm)**
1 (G+Q)	**268.1**	63.0	**68.00**
2 (G+Q)p1	**196.1**	45.0	**48.73**
6 (G+Q+Ny	**229.8**	62.6	**66.95**





Factor:	Symbol:	Value:
Coefficient – EC2, eqn. 3.15	αcc	N/A, therefore 1.0
Effective Strength – BS8110, fig. 2.1	η:	0.67
Reduction Factor – BS8110, fig. 3.3	λ:	0.9
Reduction Factor – EC2, cl.3.1.7(3)	RF:	N/A, therefore 1.0
Max. Strain of Steel	εc:	0.0035
Steel Modulus	Es:	200,000 N/mm^2
Concrete Modification Factor	γm:	1.5
Rebar Modification Factor	γms:	1.15

Plastic Centroid, yp = 200mm

Calculation:

Case G: Pure Tension from Steel

$$Total\ Axial\ Load, N = -Ast * fyd$$
$$N = -361.9kN$$

Case A: Pure Compression:

$$Total\ Axial\ Load, N = Fsc + Fcc$$
$$N = (361.91 + 2{,}131.88)kN$$
$$N = 2{,}493.79\ kN$$

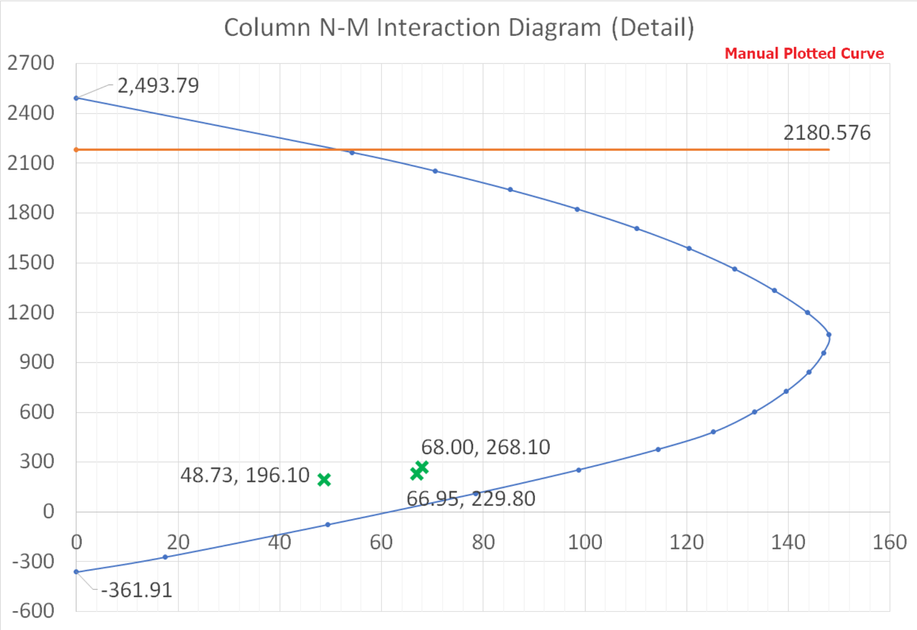
Ultimate Moment Capacity Limit: (BS8110 only)

$$N = 0.4 * fcu * Ac + 0.75 * As * fyd$$

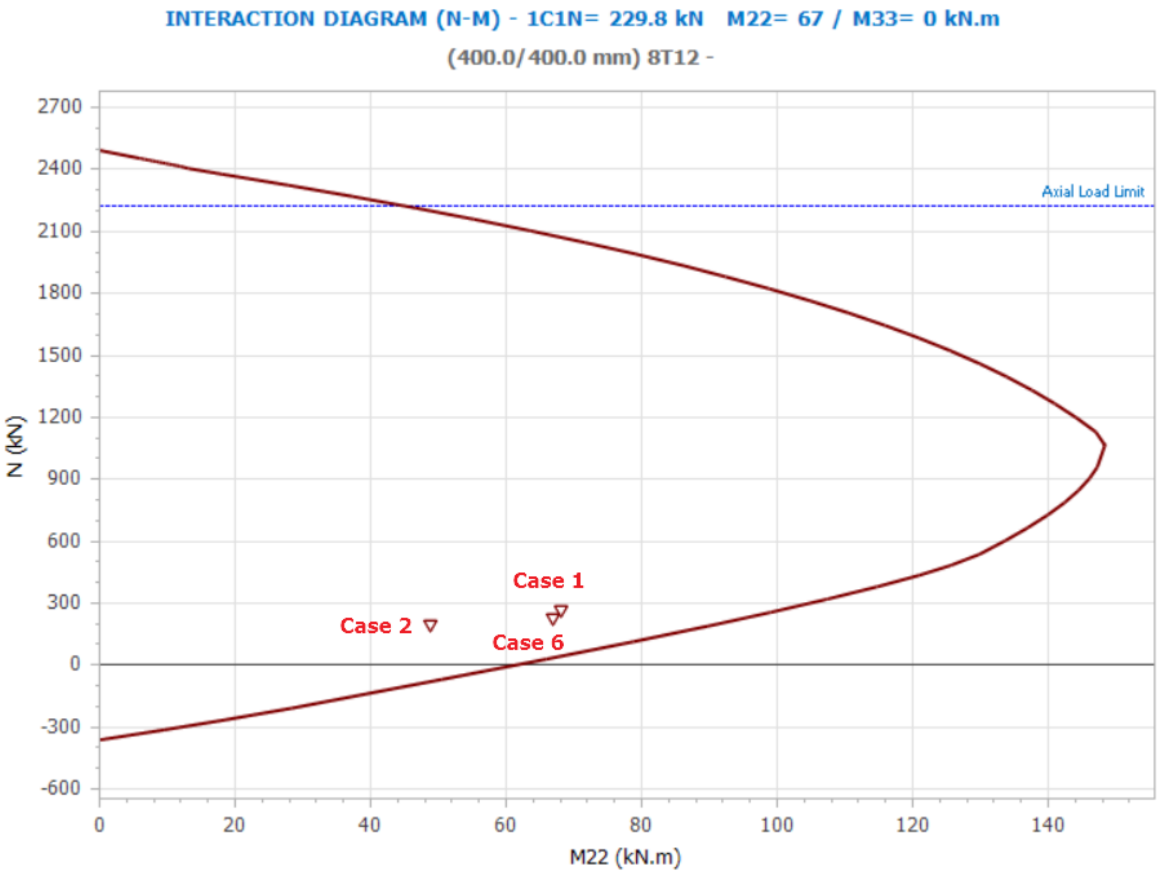
$$N = 0.4 * 30 * 160{,}000 + 0.75 * 904.8 * 400 = 2{,}180.58kN$$

The sample size for x is 20 in this example, increase the size to have higher precision of the result.

$$x\ (mm) = \begin{pmatrix} 20 \\ 40 \\ 60 \\ 80 \\ 100 \\ 120 \\ 140 \\ 160 \\ 180 \\ 200 \\ 220 \\ 240 \\ 260 \\ 280 \\ 300 \\ 320 \\ 340 \\ 360 \\ 380 \\ 400 \end{pmatrix} \quad N, ult\ (kN) = \begin{pmatrix} -272.217 \\ -75.65 \\ 111.88 \\ 253.88 \\ 377.672 \\ 481.62 \\ 600.72 \\ 725.47 \\ 843.94 \\ 958.01 \\ 1068.89 \\ 1200.27 \\ 1333.84 \\ 1462.12 \\ 1586.16 \\ 1706.75 \\ 1824.51 \\ 1939.9 \\ 2053.3 \\ 2165.01 \end{pmatrix} \quad M, ult\ (kNm) = \begin{pmatrix} 17.38 \\ 49.49 \\ 78.46 \\ 98.69 \\ 114.38 \\ 125.27 \\ 133.28 \\ 139.55 \\ 144.09 \\ 146.88 \\ 147.95 \\ 143.74 \\ 137.18 \\ 129.48 \\ 120.51 \\ 110.19 \\ 98.45 \\ 85.24 \\ 70.51 \\ 54.24 \end{pmatrix}$$



Curve for direction 3 (M22) about "plane X" from ProtaStructure is shown below:

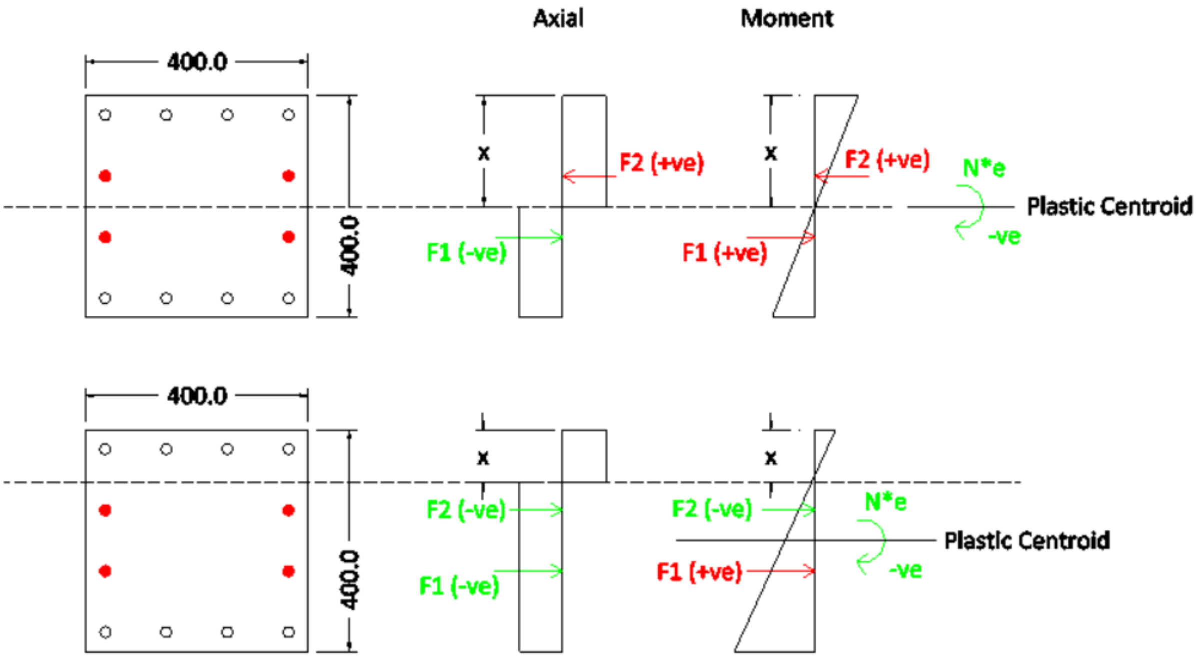


Based on the output from manually potted curve and the result generated from ProtaStructure, they are proved to be identical, hence, we can conclude that the N-M Interaction Diagram result in ProtaStructure is correct and adequate.

Intermediate Rebars Capacity

Intermediate rebars provide additional axial load capacity and moment resistance to the column/wall.

The capacity of these bars must be calculated separately and added into total column axial load and Moment capacity manually. The formula is simple and straight forward.



Formula:

Stress of a single intermediate rebar:

$$\sigma s_{,mid} = Es * \frac{\varepsilon c * (x - d_{mid})}{x} \qquad but\ -f_{yd} \leq \sigma s_{,mid} \leq f_{yd}$$

(-ve = tension, +ve = compression)

Axial load capacity of a single intermediate rebar:

$$Fs_{,mid} = \sigma s_{,mid} * As_{,mid}$$

Moment capacity of a single intermediate rebar:

$$M_{,mid} = F_{s,mid} * (yp - d_{,mid})$$

The calculated axial load capacity and moment capacity from the intermediate rebar is then summed into the column axial load capacity and moment capacity (which has not accounted for intermediate rebar capacity yet) to become the final column capacity.

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